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EML6281
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Reverse Analysis MatLab Code for PUMA Robot

My code (252 lines wow):

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% PUMA robot reverse analysis: Jackson Sammartino
% Special geometry: S1 and S2 intersect
% Special geometry: S2 and S3 are parallel
% Special geometry: S4, S5, and S6 intersect (ball and socket)
clc; clear;
% Define fixed vectors and mechanism parameters
F_S1 = [0; 0; 1];
F_x1 = [1; 0; 0];
a12 = 0; a23 = 17; a34 = 0.8; a45 = 0; a56 = 0; a67 = 0;
alpha12 = 90*pi/180; alpha23 = 0*pi/180; alpha34 = 270*pi/180;
alpha45 = 90*pi/180; alpha56 = 90*pi/180; alpha67 = 90*pi/180;
s2 = 5.9; s3 = 0; s4 = 17; s5 = 0; s6 = 4;
% Enter user inputs (inches) and unit vectors
Ptool6 = [5; 3; 7];
PtoolF = [24.112; 20.113; 18.167];
F_S6 = [0.079; -0.787; 0.612];
F_A67 = [0.997; 0.064; -0.047];
% Hypothetical closure link procedure for the 67 parameters
% Calculate some initial parameters which are the same for all 3 cases
F_y6 = cross(F_S6, F_A67);
F_P6o = PtoolF - Ptool6(1)*F_A67 - Ptool6(2)*F_y6 - Ptool6(3)*F_S6;
F_S7 = cross(F_A67, F_S6);
% Start with Case 1: S1 and S7 are not parallel and not colinear (loose tol)
try
    F_A71 = cross(F_S7, F_S1) / norm(cross(F_S7, F_S1));
    % gamma1 must satisfy both cosine and sine equations (loose tol)
    gamma1 = acos(dot(F_A71, F_x1));
    if abs(sin(gamma1) - dot(cross(F_A71, F_x1), F_S1)) > 10^-1
        gamma1 = asin(dot(cross(F_A71, F_x1), F_S1));
    end
    % theta7 must satisfy both cosine and sine equations (loose tol)
    theta7 = acos(dot(F_A67, F_A71));
    if abs(sin(theta7) - dot(cross(F_A67, F_A71), F_S7)) > 10^-1
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        theta7 = asin(dot(cross(F_A67, F_A71), F_S7));
    end
    % alpha71 must satisfy both cosine and sine equations (loose tol)
    alpha71 = acos(dot(F_S7, F_S1));
    if abs(sin(alpha71) - dot(cross(F_S7, F_S1), F_A71)) > 10^-1
        alpha71 = asin(dot(cross(F_S7, F_S1), F_A71));
    end
    % save alpha71 for other cases
    alphasave = alpha71;
    s7 = dot(cross(F_S1, F_P6o), F_A71) / sin(alpha71);
    a71 = dot(cross(F_P6o, F_S1), F_S7) / sin(alpha71);
    s1 = dot(cross(F_P6o, F_S7), F_A71) / sin(alpha71);
catch
    % Special Cases 2 and 3: S1 and S7 are parallel (loose tol)
    if norm(cross(F_S1, F_S7)) < 10^-1
        s1 = -F_P6o*F_S1;
        a71 = norm(dot(-F_P6o, s1*F_S1));
        % Special Case 3: S1 and S7 are also colinear (loose tol)
        if a71 < 10^-1
            s1 = -F_P6o*F_S1;
            a71 = 0;
            theta7 = 0;
            s7 = 0;
            alpha71 = alphasave;
            gamma1 = acosdot(F_A71, F_x1);
            if abs(sin(gamma1) - dot(cross(F_A71, F_x1), F_S1)) > 10^-1
                gamma1 = asin(dot(cross(F_A71, F_x1), F_S1));
            end
        end
        F_A71 = -(F_P6o + s1*F_S1) / a71;
        alpha71 = alphasave;
        s7 = 0;
        theta7 = acos(dot(F_A67, F_A71));
        if abs(sin(theta7) - dot(cross(F_A67, F_A71), F_S7)) > 10^-1
            theta7 = asin(dot(cross(F_A67, F_A71), F_S7));
        end
        gamma1 = acos(F_A71, F_x1);
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        if abs(sin(gamma1) - dot(cross(F_A71, F_x1), F_S1)) > 10^-1
            gamma1 = asindot(cross(F_A71, F_x1), F_S1);
        end
    end
end
% Reverse analysis procedure to solve for theta1 through theta6
% 2 possible values for theta1 (use trig method)
X7 = sin(alpha67)*sin(theta7);
Y7 = -1*(sin(alpha71)*cos(alpha67) + cos(alpha71)*sin(alpha67)*cos(theta7));
Z7 = cos(alpha71)*cos(alpha67) - sin(alpha71)*sin(alpha67)*cos(theta7);
A = s6*Y7 - s7*sin(alpha71);
B = s6*X7 + a71;
D = s2;
gamma1 = acos(A/sqrt(A^2 + B^2));
if abs(sin(gamma1) - B/sqrt(A^2 + B^2)) > 0.01
    gamma1 = asin(B/sqrt(A^2 + B^2));
end
theta1a = gamma1 + acos(-D/sqrt(A^2 + B^2));
theta1b = gamma1 - acos(-D/sqrt(A^2 + B^2));
theta1 = [theta1a theta1b];
% 4 possible values for theta3 (trig method)
theta3 = zeros(1, 4);
for ii = 1:1:length(theta1)
    X1 = sin(alpha71)*sin(theta1(ii));
    Y1 = -1*(sin(alpha12)*cos(alpha71) + cos(alpha12)*sin(alpha71)*cos(theta1(ii)));
    X71 = X7*cos(theta1(ii)) - Y7*sin(theta1(ii));
    Y71 = cos(alpha12)*(X7*sin(theta1(ii)) + Y7*cos(theta1(ii))) - sin(alpha12)*Z7;
    a = -s6*X71 - s7*X1 - a71*cos(theta1(ii));
    b = s1 - s6*Y71 - s7*Y1;
    A = 2*a23*a34;
    B = -2*a23*s4;
    D = a23^2 + a34^2 + s4^2 - a^2 - b^2;
    gamma3 = acos(A/sqrt(A^2 + B^2));
    if abs(sin(gamma3) - B/sqrt(A^2 + B^2)) > 0.01
        gamma3 = asin(B/sqrt(A^2 + B^2));
    end
    theta3(ii) = gamma3 + acos(-D/sqrt(A^2 + B^2));
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    theta3(ii+1) = gamma3 - acos(-D/sqrt(A^2 + B^2));
end
% 4 possible values for theta2
theta2 = zeros(1, 4);
for ii = 1:1:length(theta1)
    for jj = 1:1:length(theta3)/2
        X1 = sin(alpha71)*sin(theta1(ii));
        Y1 = -1*(sin(alpha12)*cos(alpha71) + cos(alpha12)*sin(alpha71)*cos(theta1(ii)));
        X71 = X7*cos(theta1(ii)) - Y7*sin(theta1(ii));
        Y71 = cos(alpha12)*(X7*sin(theta1(ii)) + Y7*cos(theta1(ii))) - sin(alpha12)*Z7;
        a = -s6*X71 - s7*X1 - a71*cos(theta1(ii));
        b = s1 - s6*Y71 - s7*Y1;
        ratio = (-a34*sin(theta3(jj)) - s4*cos(theta3(jj)))/(a23 + a34*cos(theta3(jj)) - s4*sin(theta3(jj)));
        tan2 = (a - 1)/ratio;
        value2 = atan(tan2);
        if ii == 1
            theta2(jj) = value2;
        elseif ii == 2
            theta2(jj+1) = value2;
        end
    end
end
% 8 possible values for theta5
theta5 = zeros(1, 8);
for ii = 1:1:8
    if ii >= 4
        i1 = 1;
    else
        i1 = 2;
    end
    t1 = theta1(i1);
    i3or2 = ceil(ii/2);
    t2 = theta2(i3or2);
    t3 = theta3(i3or2);
    t7 = theta7;
    X1 = sin(alpha71)*sin(t1);
    Y1 = -1*(sin(alpha12)*cos(alpha71) + cos(alpha12)*sin(alpha71)*cos(t1));
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    Z1 = cos(alpha12)*cos(alpha71) - sin(alpha12)*sin(alpha71)*cos(t1);
    X2 = sin(alpha12)*sin(t2);
    Y2 = -1*(sin(alpha23)*cos(alpha12) + cos(alpha23)*sin(alpha12)*cos(t2));
    Z2 = cos(alpha23)*cos(alpha12) - sin(alpha23)*sin(alpha12)*cos(t2);
    X3 = sin(alpha23)*sin(t3);
    Y3 = -1*(sin(alpha34)*cos(alpha23) + cos(alpha34)*sin(alpha23)*cos(t3));
    Z3 = cos(alpha34)*cos(alpha23) - sin(alpha34)*sin(alpha23)*cos(t3);
    X3bar = sin(alpha34)*sin(t3);
    Y3bar = -1*(sin(alpha23)*cos(alpha34) + cos(alpha23)*sin(alpha34)*cos(t3));
    Z3bar = cos(alpha23)*cos(alpha34) - sin(alpha23)*sin(alpha34)*cos(t3);
    X7 = sin(alpha67)*sin(t7);
    Y7 = -1*(sin(alpha71)*cos(alpha67) + cos(alpha71)*sin(alpha67)*cos(t7));
    Z7 = cos(alpha71)*cos(alpha67) - sin(alpha71)*sin(alpha67)*cos(t7);
    X32 = X3bar*cos(t2) - Y3bar*sin(t2);
    Y32 = cos(alpha12)*(X3bar*sin(t2) + Y3bar*cos(t2)) - sin(alpha12)*Z3bar;
    Z32 = sin(alpha12)*(X3bar*sin(t2) + Y3bar*cos(t2)) + cos(alpha12)*Z3bar;
    X321 = X32*cos(t1) - Y32*sin(t1);
    Y321 = cos(alpha71)*(X32*sin(t1) + Y32*cos(t1)) - sin(alpha71)*Z32;
    Z321 = sin(alpha71)*(X32*sin(t1) + Y32*cos(t1)) + cos(alpha71)*Z32;
    Z3217 = sin(alpha67)*(X321*sin(t7) + Y321*cos(t7) + cos(alpha67)*Z321);
    theta5(ii) = acos(-1*Z3217);
end
% 8 possible values for theta4
theta4 = zeros(1, 8);
for ii = 1:1:8
     if ii >= 4
        i1 = 1;
    else
        i1 = 2;
    end
    t1 = theta1(i1);
    i3or2 = ceil(ii/2);
    t2 = theta2(i3or2);
    t3 = theta3(i3or2);
    t5 = theta5(ii);
    t7 = theta7;
   X1 = sin(alpha71)*sin(t1);
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    Y1 = -1*(sin(alpha12)*cos(alpha71) + cos(alpha12)*sin(alpha71)*cos(t1));
    Z1 = cos(alpha12)*cos(alpha71) - sin(alpha12)*sin(alpha71)*cos(t1);
    X2 = sin(alpha12)*sin(t2);
    Y2 = -1*(sin(alpha23)*cos(alpha12) + cos(alpha23)*sin(alpha12)*cos(t2));
    Z2 = cos(alpha23)*cos(alpha12) - sin(alpha23)*sin(alpha12)*cos(t2);
    X3 = sin(alpha23)*sin(t3);
    Y3 = -1*(sin(alpha34)*cos(alpha23) + cos(alpha34)*sin(alpha23)*cos(t3));
    Z3 = cos(alpha34)*cos(alpha23) - sin(alpha34)*sin(alpha23)*cos(t3);
    X7 = sin(alpha67)*sin(t7);
    Y7 = -1*(sin(alpha71)*cos(alpha67) + cos(alpha71)*sin(alpha67)*cos(t7));
    Z7 = cos(alpha71)*cos(alpha67) - sin(alpha71)*sin(alpha67)*cos(t7);
    X71 = X7*cos(t1) - Y7*sin(t1);
    Y71 = cos(alpha12)*(X7*sin(t1) + Y7*cos(t1)) - sin(alpha12)*Z7;
    Z71 = sin(alpha12)*(X7*sin(t1) + Y7*cos(t1)) + cos(alpha12)*Z7;
    X712 = X71*cos(t2) - Y71*sin(t2);
    Y712 = cos(alpha23)*(X71*sin(t2) + Y71*cos(t2)) - sin(alpha23)*Z71;
    X7123 = X712*cos(t3) - Y712*sin(t3);
    % free choice for theta4 to avoid division by zero
    if abs(sin(t5)) < 0.001
        theta4(ii) = 2;
    else
        theta4(ii) = acos(X7123)/sin(t5);
    end
end
% 8 possible values for theta6
theta6 = zeros(1, 8);
for ii = 1:1:8
     if ii >= 4
        i1 = 1;
    else
        i1 = 2;
    end
    t1 = theta1(i1);
    i3or2 = ceil(ii/2);
    t2 = theta2(i3or2);
    t3 = theta3(i3or2);
    t4 = theta4(ii);
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    t5 = theta5(ii);
    t7 = theta7;
    X1 = sin(alpha71)*sin(t1);
    Y1 = -1*(sin(alpha12)*cos(alpha71) + cos(alpha12)*sin(alpha71)*cos(t1));
    Z1 = cos(alpha12)*cos(alpha71) - sin(alpha12)*sin(alpha71)*cos(t1);
    X2 = sin(alpha12)*sin(t2);
    Y2 = -1*(sin(alpha23)*cos(alpha12) + cos(alpha23)*sin(alpha12)*cos(t2));
    Z2 = cos(alpha23)*cos(alpha12) - sin(alpha23)*sin(alpha12)*cos(t2);
    X3 = sin(alpha23)*sin(t3);
    Y3 = -1*(sin(alpha34)*cos(alpha23) + cos(alpha34)*sin(alpha23)*cos(t3));
    Z3 = cos(alpha34)*cos(alpha23) - sin(alpha34)*sin(alpha23)*cos(t3);
    X4bar = sin(alpha45)*sin(t4);
    Y4bar = -1*(sin(alpha34)*cos(alpha45) + cos(alpha34)*sin(alpha45)*cos(t4));
    Z4bar = cos(alpha34)*cos(alpha45) - sin(alpha34)*sin(alpha45)*cos(t4);
    X7 = sin(alpha67)*sin(t7);
    Y7 = -1*(sin(alpha71)*cos(alpha67) + cos(alpha71)*sin(alpha67)*cos(t7));
    Z7 = cos(alpha71)*cos(alpha67) - sin(alpha71)*sin(alpha67)*cos(t7);
    X43 = X4bar*cos(t3) - Y4bar*sin(t3);
    Y43 = cos(alpha23)*(X4bar*sin(t3) + Y4bar*cos(t3)) - sin(alpha23)*Z4bar;
    Z43 = sin(alpha23)*(X4bar*sin(t3) + Y4bar*cos(t3)) + cos(alpha23)*Z4bar;
    X432 = X43*cos(t2) - Y43*sin(t2);
    Y432 = cos(alpha12)*(X43*sin(t2) + Y43*cos(t2)) - sin(alpha12)*Z43;
    Z432 = sin(alpha12)*(X43*sin(t2) + Y43*cos(t2)) + cos(alpha12)*Z43;
    X4321 = X432*cos(t1) - Y432*sin(t1);
    Y4321 = cos(alpha71)*(X432*sin(t1) + Y432*cos(t1)) - sin(alpha71)*Z432;
    X43217 = X4321*cos(t7) - Y4321*sin(t7);
    theta6(ii) = asin(X43217);
end
% format and print all cases for all 7 angles
table = {theta1, theta2, theta3, theta4, theta5, theta6, theta7};
for ii = 1:1:numel(table)
    fprintf('Theta %d: ', ii);
    fprintf('%g, ', table{ii});
    fprintf('\n');
end
```

Output (in radians):

Theta 1: 4.19308, 0.616398,
Theta 2: -1.55071, 1.55216, 1.55216, 0,
Theta 3: -0.571559, -0.571559, -2.47599, 0,
Theta 4: 0.661119, 0.661119, 2.50207, 4.91475, 0.403655, 0.403655, 0.966653, 0.966653,
Theta 5: 1.45006, 1.45006, 1.6606, 0.348351, 1.77935, 1.77935, 0.912592, 0.912592,
Theta 6: 0.27133, 0.27133, -0.270062, 0.0995357, -1.15216, -1.15216, -1.06088, -1.06088,
Theta 7: -0.076673,
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